

KINETIC DROPLET EVAPORATION IN A TURBULENT AIR JET

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Droplet evaporation in a turbulent air jet is considered for conditions such that the evaporation rate is determined by the evaporation kinetics for the individual droplets rather than by the diffusion rate of the air in the jet. Numerical solution of the equations by computer has shown that the mean air speed in the jet has little effect on the droplet evaporation in the range covered. A simplified solution is presented for the dispersal of an evaporating impurity in a jet. Experiments confirm that this solution is correct, and they show that the turbulent pulsations play a large part, with the result that an initially monodisperse system becomes more and more polydisperse away from the jet.

Technical studies on droplet evaporation are usually based on measurements on individual droplets; not much is known about the actual evaporation processes for droplet systems, especially ones suspended in turbulent gas jets.

It has been shown [1] that the evaporation rate for a droplet system in a turbulent air jet may be determined by the evaporation kinetics of the individual droplets (kinetic state) or by the diffusion rate of the outer air into the jet as a whole (diffusion rate). The state of evaporation can be defined via a criterion that characterizes the ratio of the individual droplet evaporation time τ_1 to the time τ_2 spent in the internal zone, where the air is sufficient for all the droplets to evaporate completely, $E = \tau_1/\tau_2$; if $E \gg 1$, one gets the kinetic state, while the diffusion state occurs for $E \ll 1$.

Here we consider evaporation in the kinetic state.

The droplets evaporate in a turbulent air jet because they move relative to the air, on account of the difference in speed between the droplet and the mean motion of the air, together with the turbulent pulsations.

The following formula incorporates the effects of the average air speed on the rate of reduction in the radius r of an evaporating drop [2]:

$$dr = - (D / \rho_1 r) (c_0 - c_\infty) (1 + \beta \text{Re}^{1/2} \text{Sc}^{1/3}) d\tau, \quad \text{Re} = 2(v - u) r / \nu \quad (1)$$

where D is the diffusion coefficient of the vapor in air, c_0 is the vapor concentration at saturation at the temperature at the droplet surface, c_∞ is the vapor concentration in the surrounding air, ρ_1 is the density of the liquid, Re is Reynolds number, $\text{Sc} = \nu/D$ is Smith's criterion, τ is time, v is droplet speed, u is air speed, ν is kinematic viscosity for air, and $\beta \approx 0.30$.

The equation of motion for a droplet of variable mass is

$$d(mv)/d\tau = \psi S \rho_2 (v - u)^2 / 2$$

or as follows after the substitutions $m = 4/3 \pi r^3 \rho_1$, $S = \pi r^2$

$$\frac{4}{3} \rho_1 \left(r \frac{dv}{d\tau} + 3v \frac{dr}{d\tau} \right) = \psi \rho_2 \frac{(v - u)^2}{2} \quad (2)$$

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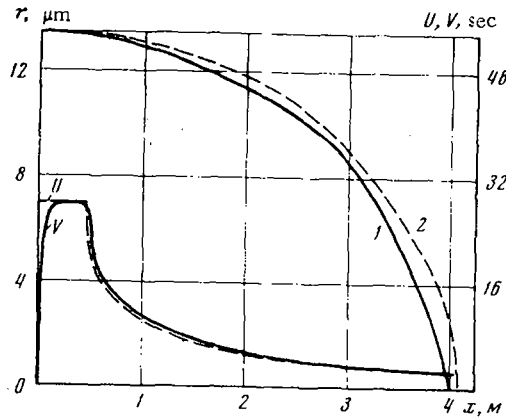


Fig. 1

where m is droplet mass, S is the area of the middle cross section, ρ_2 is the density of air, and ψ is the droplet resistance coefficient.

It is stated [3] that the latter for $6 \leq Re \leq 400$ is

$$\psi \approx 0.12 + 37 / Re \quad (3)$$

The following is the air speed at the axis of the initial part of a turbulent free jet of circular cross section [4]:

$$u = \text{const} = u_0 \quad (4)$$

while that in the main part of the jet is

$$u = 12.4 R_0 u_0 x^{-1} (1 - \xi^{1.5})^2 \quad (5)$$

where R_0 is the radius of the initial cross section and x is the distance from that cross section:

$$\xi = 4.2y / x \quad (6)$$

y being the distance from the jet axis.

In the range of temperatures for water $T_k = 283-293^\circ \text{K}$

$$c_0 \approx 7.9 \cdot 10^{-7} T_k - 2.148 \cdot 10^{-4} \quad (7)$$

where c_0 is in g/cm^3 .

Maxwell [2] gave as follows for the quasistationary evaporation of a drop in immobile air:

$$T_\infty - T_h = DL\lambda^{-1} (c_0 - c_\infty) \quad (8)$$

where λ is the thermal conductivity of air and L is the latent heat of evaporation.

From Eqs. (7) and (8) we have for water

$$c_0 \approx \frac{7.9 \cdot 10^{-7} (T_\infty + DL\lambda^{-1} c_\infty) - 2.148 \cdot 10^{-4}}{1 + 7.9 \cdot 10^{-7} DL\lambda^{-1}} \quad (9)$$

where T_∞ is the temperature of the surrounding air.

Equations (1)-(9) have been solved by computer for droplets of water and ethanol; it was assumed that the drop moves along the axis of a turbulent air jet ($\xi = 0$) from the point $x=0$, where the radius $r=r_0$ and velocity $v=0$. Solutions were obtained for r_0 from 10 to 55 μ , and u_0 from 28 to 150 m/sec, with R_0 from 3 to 7 cm, c_∞ from 0 to $9 \cdot 10^{-6} \text{ g/cm}^3$, c_0 from $9.4 \cdot 10^{-6}$ to $53 \cdot 10^{-6} \text{ g/cm}^3$, D from 0.135 to 0.250 cm^2/sec , L from 222 to 591 cal/g, $\lambda = 6.2 \cdot 10^{-5} \text{ cal/deg} \cdot \text{cm} \cdot \text{sec}$, and $\rho_2 = 1.2 \cdot 10^{-3} \text{ g/cm}^3$.

The solid line in Fig. 1 shows typical results (r for various distances x from the nozzle); it also shows the results for $r = f(x)$ from the simplified formula

$$r = \{r_0^2 - Dx^2(c_0 - c_\infty) / 12.4\rho_1 R_0 u_0\}^{1/2} \quad (10)$$

(broken line), as obtained from Eq. (1) by neglecting the effects of droplet motion relative to the air ($\beta Re^{1/2} Sc^{1/3} = 0$, $d\tau = dx/u$).

The values for x_m , the distance traveled by the drop along the axis before complete evaporation, as calculated from Eq. (10) are larger than those found by numerical solution, but the differences were only 1.5 to 7% in the above ranges in the parameters, i.e., one can neglect the difference between the drop speed and mean air speed in approximate calculations in this range and thus use Eq. (10).

Figure 2 shows a scheme for an air and droplet jet in the kinetic mode; the broken line denotes the boundary between the core of the jet in which the droplets of identical initial radius r_0 have not completely evaporated and the periphery of the jet, where there are no droplets. This boundary is defined by Eq. (10) with $r = 0$:

$$\begin{aligned} x &= 3.52r_0 \sqrt{\frac{R_0 u_0 \rho_2}{D(c_0 - c_\infty)}} (1 - \xi^{1.5}) = x_m (1 - \xi^{1.5}), \\ x_m &= 3.52r_0 \sqrt{\frac{R_0 u_0 \rho_2}{D(c_0 - c_\infty)}} \end{aligned} \quad (11)$$

where x_m is the length of the core.

Figure 2 clearly illustrates the dispersal of the evaporating droplets, which differs from that of conserved impurity.

In the case of evaporating impurity, the amount of liquid in the jet decreases as x increases and becomes zero for $x = x_m$, $\xi = 0$; a conserved impurity is constant in amount at all x . In the latter case, the distributions for dimensionless concentrations are of the same shape in all sections, whereas this is not so for an evaporated impurity.

This means that the solutions obtained in jet theory for dispersal of a conserved impurity [4] are unsuitable for a jet with a evaporating impurity; the method used in jet theory to solve the dispersal problem is based on referring the diffusion equation in partial derivatives with variables x and ξ to an ordinary differential equation with one variable ξ , which is possible on account of the similarity in the dimensionless profiles. This is clearly inapplicable for the case of evaporation.

In the latter case, the droplet concentration n is found by solving the diffusion equation [4]:

$$u \frac{\partial n}{\partial x} = \frac{\partial}{\partial \xi} \left(K \frac{\partial n}{\partial \xi} \right) \quad \text{for } u = u_m (1 - \xi^{1.5})^2, \quad K = l_T^2 \frac{\partial u}{\partial y} \sim x^2 \frac{\partial u}{\partial y}$$

and is defined by Eq. (11) with the boundary condition $n = 0$ on the axially symmetrical surface with the generator $x = f(\xi)$.

As the solution is cumbersome, we restrict ourselves to the following approximation. Drops displaced from the core by the turbulent pulsations evaporate and do not return to the core.

We assume that the concentration is not zero but has a certain constant value n_3 at the surface of the core in a jet with the generator of Eq. (11); then in one set there is the following reduction in the number of drops in the part of the core of extent x in 1 sec:

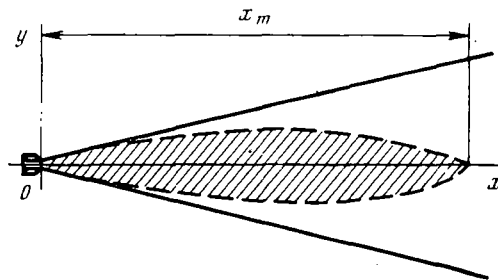


Fig. 2

$$\Delta N = -n_S \int_0^x av' 2\pi y dx \quad (12)$$

where av' is the mean transverse velocity of the droplets, which we take as proportional to the mean transverse velocity pulsation of the air v' .

Then n_S is defined by the normalization condition:

$$n_S \int_0^{x_m} av' 2\pi y dx = N_0 \quad (13)$$

where N_0 is the total number of drops passing through the initial cross section of the jet in 1 sec.

It is found [4] that

$$v' \approx u' \approx l_T \partial u / \partial y = Cx \partial u / \partial y \quad (14)$$

where l_T is the "mixing length" and C is a constant.

We express v' and y in terms of the coordinate x of the jet core as defined by Eq. (11); from Eqs. (5) and (6) we get

$$y = \frac{x}{4.2} \left(1 - \frac{x}{x_m}\right)^{1/2}, \quad v' = -\frac{156R_0u_0C}{x_m} \left(1 - \frac{x}{x_m}\right)^{1/2}$$

and substitute these expressions into Eq. (12) to get

$$\Delta N = 2\pi n_S a \int_0^x \frac{156R_0u_0C}{x_m} \left(1 - \frac{x}{x_m}\right)^{1/2} \frac{x}{4.2} \left(1 - \frac{x}{x_m}\right)^{1/2} dx = A \left(\frac{x^2}{2x_m} - \frac{x^3}{3x_m^2}\right)$$

Similarly, we transform Eq. (13) to $N_0 = Ax_m/6$.

The following is the relative number of droplets passing through the cross section of coordinate x in 1 sec:

$$N / N_0 = 1 - \Delta N / N_0 = 1 - 3(x/x_m)^2 + 2(x/x_m)^3 \quad (15)$$

This formula indicates that the flux N of droplets, which equals N_0 in the initial cross section, falls to half at $x = x_m/2$ and vanishes at $x = x_m$.

Our experiments were designed to establish the effects of a major factor not incorporated in droplet evaporation theory, namely the turbulent velocity pulsation in the jets.

The apparatus was in a room of size $10 \times 5.7 \times 2.8$ m³; the air was blown in by a fan through a meter to a circular nozzle, whose axis was horizontal. Droplets of uniform size were produced from distilled water by a rotating system (an air turbine with an air bearing, having the liquid fed via a needle to the center of the rotor and a small adjustable gap between the needle and the rotor [5]). This source was before the nozzle. The droplets were thrown outwards from a slot in the body as a flat divergent stream directed away from the jet axis, i.e., unsymmetrically. We found that the distribution became axially symmetrical at $x = 50$ cm for droplets of initial radius $r_0 = 14$ μ , the distribution being close to the theoretical one for a conserved impurity. The initial droplet size was determined by extraction into a cell containing an immersion medium (a mixture of spindle and other oils) at $x = 30$ cm from the nozzle, with subsequent measurement of the droplets under the microscope.

We determined r and n_1 at points 1.9 to 7 m from the nozzle, using deposition in strips coated with a film of 10% solution of Collargol in water [6]. The droplets formed light circular impressions on the film, which were measured under the microscope. We found that the spreading factor was 2.44 for radii of 13-43 μ at speeds of 1-6 m/sec (coefficient of variation 3.5%). The trapping factor was calculated from the data of [7]. The exposure times varied from 60 to 1320 sec. We also calculated the theoretical concentrations n_2 from the formulas for turbulent jets with conserved materials.

Cascade impactors [8] were also used to sample the droplets at the axis, using glass plates coated with Collargol.

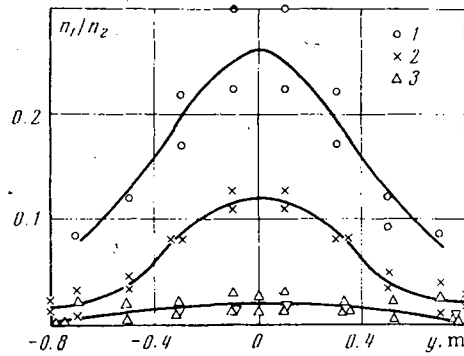


Fig. 3

The experimental jet coefficient [4] was found as 0.0764 from the velocity profiles; the initial droplet concentrations were very small (less than 1 cm^{-3}), while E was large (about 10^8), i.e., the mode of evaporation was kinetic.

Table 1 gives results from the deposition on the strips, which are also represented by the typical curves of Fig. 3, which give the relative droplet concentrations (n_1/n_2) at various distances for $r_0 = 20 \mu$. The points denoted by 1 in Fig. 3 relate to $x = 3.5 \text{ m}$, while those denoted by 2 relate to 5.0 m, and those by 3 to 6.0 m.

The values found for n_1 were much less than for n_2 ; n_1/n_2 decreased rapidly away from the nozzle (as x increased) and also away from the axis (as y increased).

The values of N/N_0 calculated from Eq. (15) were compared with those found by experiment, and it was found that most of the observed points lay below the theoretical curve, which was due to the excessive values assumed for N_0 in the calculations (some of the droplets formed by the source did not enter the jet). The agreement between theory and experiment is to be considered satisfactory in view of the accuracy of the measurements.

We were able to relate the standard deviation σ of the droplet radii at a particular point to the mean r for various distances from the nozzle for r_0 of 14 and 20μ ; the value of σ near the nozzle was about 5%, but away from it it rose rapidly and varied from 8.5 to 17.5% at 1.7 m and from 23.2 to 28% at 4.7 m. Then an initially monodisperse droplet system evaporating in a turbulent jet becomes more polydisperse as the nozzle is left behind; this is due to the turbulent velocity pulsations, which are not incorporated in Eq. (1), which would indicate that an initially monodisperse aerosol would remain the same if only the mean air speeds were involved.

Figure 4 shows the variation in the mean droplet radius r along the axis given by the measurements (1 for $r_0 = 14 \mu$, 2 for $r_0 = 20 \mu$, and 3 for $r_0 = 25 \mu$). The solid lines represent numerical solutions from Eqs. (1)-(9). Initially the measured r fall more rapidly than the theoretical ones, which is ascribed to the neglected effects of the turbulent velocity pulsations. Then at large distances from the nozzle the measured and theoretical r become identical, after which the theoretical r decrease more rapidly than the measured ones down to complete evaporation ($r=0$). The reason for this will be clear from Fig. 5, which shows the fall in n_1/n_2 along the axis (points 1 correspond to $r_0 = 14 \mu$, 2 to 20μ , and 3 to 25μ).

The droplet concentration at the axis was about 30% of the theoretical value n_2 for a conservative material at $x = 1.9 \text{ m}$, and the value at larger x fell rapidly to 1% or less of n_2 . This has been explained above

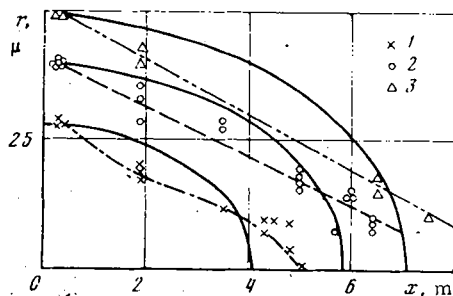


Fig. 4

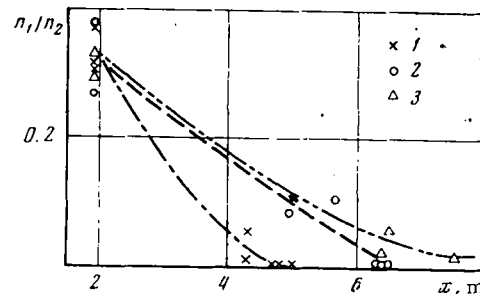


Fig. 5

TABLE 1

No.	Relative humidity, %	Air temperature, °C	r_b , μ	x , μ	Mean droplet radius (μ) deposited on strips											
					η_1/η_2											
					U, μ											
					0.05	0.10	0.30	0.50	0.70	0.80	0.05	0.10	0.30	0.50	0.70	0.80
1	25	25.8	42	2.00 3.50 6.00	42 29 35	37 29 31	36 29 32	35 29 28	35 25 28	33 27	0.625 0.21 0.60	0.435 0.21 0.437	0.357 0.209 0.236	0.271 0.185 0.189	0.185 0.185 0.435	0.588 0.182
2	33	25.0	37	3.50 6.00	23 20	23 18	23 20	22 19	21 19	22 18	0.286 0.45	0.289 0.874 0.49	0.218 0.74 0.435	0.233 0.63 0.33	0.213 0.613 0.372	0.588 0.182
3	22	24.4	36	3.50 6.00	23 20	23 18	23 20	22 19	21 19	22 18	0.014	0.009 0.125 0.125	0.081 0.081 0.081	0.041 0.041 0.041	0.017 0.017 0.017	0.204 0.022 0.0264
4	25	23.6	21	5.00	7	10	9	8	8	6	0.03	0.305	0.22	0.12	0.09	0
5	27	25.2	20	6.00	7	14	14	13	12	7	0.03	0.03	0.02	0.02	0.03	0
6	32	23.6	20	3.50 6.00	8 7	8 7	8 7	8 7	7 7	7	0.008	0.012	0.008	0.007	0.013	0
7	34	23.7	20	3.50 6.00	6 5	6 5	6 5	6 6	6 6	6	0.071	0.066	0.053	0.029	0.013	0
8	29	25.0	43	4.00 4.50	5 5	5 5	5 5	5 5	5 5	5	0.008	0.011	0	0.0083	0	0

in deriving Eq. (15); there is irreversible loss of droplets from the core, which is reflected in Eq. (15), and this is accentuated by the statistical character of evaporation arising from the turbulent velocity fluctuations, though droplets identical in r_0 in the initial cross section move to point x along different paths and therefore different radii at that point. The measured values for the mean r shown in Fig. 4 for large x consequently represent not the original set of droplets but some small number of droplets of elevated initial radius, which have passed through conditions least favorable to evaporation.

Then the curves of Figs. 3-5 not only confirm the proposed theory for an evaporating impurity in a turbulent jet (dispersal accelerated relative to that for a conserved impurity) but also illustrate an important feature of droplet evaporation in a turbulent jet: the stochastic character of the process, which transforms a monodisperse system into a polydisperse one not only over the cross section of the jet as a whole but also at each point.

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